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AN OPTIMUM INTERCEPTION LAW WITH BOUNDED CONTROL IN PRESENCE OF NOISE

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FOREWORD

This report was prepared by the Department of Electrical Engineering of the University of Southern California under USAF Contract No. F 04(695)-67-C-0109. This contract was initiated under Project No. 5218 University Program, Task No. 10, "Aerospace Vehicle Detection and Tracking Systems". The work was administered under the direction of Space Systems Division, Air Force Systems Command with Lt. Amoroso acting as project officer and technical support furnished through the TDPS Office of Aerospace Corporation.

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ABSTRACT

The subject of this report is the derivation of optimum control rule for guidance of a space vehicle which seeks to reach a moving target. The available thrust vector is assumed to be bounded in magnitude and any on-board measurements to be contaminated by random noise. The performance measure is chosen to be the expectation of terminal miss distance. The optimum control rule is shown to be a function of the sign of the expected value of line-of-sight angular rotation with respect to an inertial coordinate system and, consequently, measurement of this angular rate (e.g., by means of an angle tracking system) is sufficient to implement the optimum control rule. The optimum controller is very simple and more suited to space application than the conventional method of proportional navigation commonly used in missile systems operating in lower altitudes due to the desirability of using reaction jets in space vehicles.

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I. INTRODUCTION

This study is concerned with techniques for control of an interceptor which seeks to reach a moving target in space. When the control of the interceptor is based upon data that is contaminated with noise, and when both the interceptor and the target are assumed to be point masses, the probability of interception will, in general, be zero and the distance of closest approach will be a random variable. In such a circumstance we may use the expected value of the miss distance as a performance measure.

If the target trajectory is available to the interceptor either through a priori knowledge or by some on-board measurements, the interception problem may be solved by determining the time and point of closest approach and choosing a controller which satisfies terminal condition. The control may be of an open-loop nature. When the interceptor dynamics are governed by linear differential equations with bounded control the solution to the problem is discussed in [1] and [2]. However, in practice the amount of information concerning target motion is inadequate to enable the controller to generate a reasonable estimate of target trajectory since, at any given time, the future target acceleration is unknown. Furthermore, it may not be desirable or practical to measure the interceptor-target range accurately. Consequently, it is clear that a closed loop solution to the problem should be sought. This is an old problem and has been solved in practice by means of choosing a reasonable form for the interceptor control law and then minimizing the expected miss distance through

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simulation by adjusting various parameters. A very common scheme is referred to as proportional navigation in which the interceptor attempts to reduce the rotation of the line-of-sight to zero by applying control to change the direction of its velocity vector (it is called proportional navigation because the component of interceptor acceleration normal to its velocity vector is made proportional to the line-of-sight angular rate. In a recent paper [3] it was shown that this scheme is optimum under three very restrictive conditions, namely; (1) in the absence of any noise, (2) having the a priori knowledge of flight duration, and (3) when the performance criterion is taken to be a quadratic function of the terminal miss and the control vector where the control vector is assumed unbounded. Although this is an interesting result, yet, it does not yield the solution to the practical problem cited above. In this work an optimum closed loop solution to the problem is derived where the control is bounded in magnitude and the performance measure is chosen to be the mathematical expectation of the terminal miss distance.

II. DERIVATION OF EQUATION OF MOTION

For this study the planar motion of the target and the interceptor will be considered. The solution to the three dimensional problem may be obtained in a similar manner. Figure 1 displays the target-interceptor geometry with respect to a target centered coordinate system.

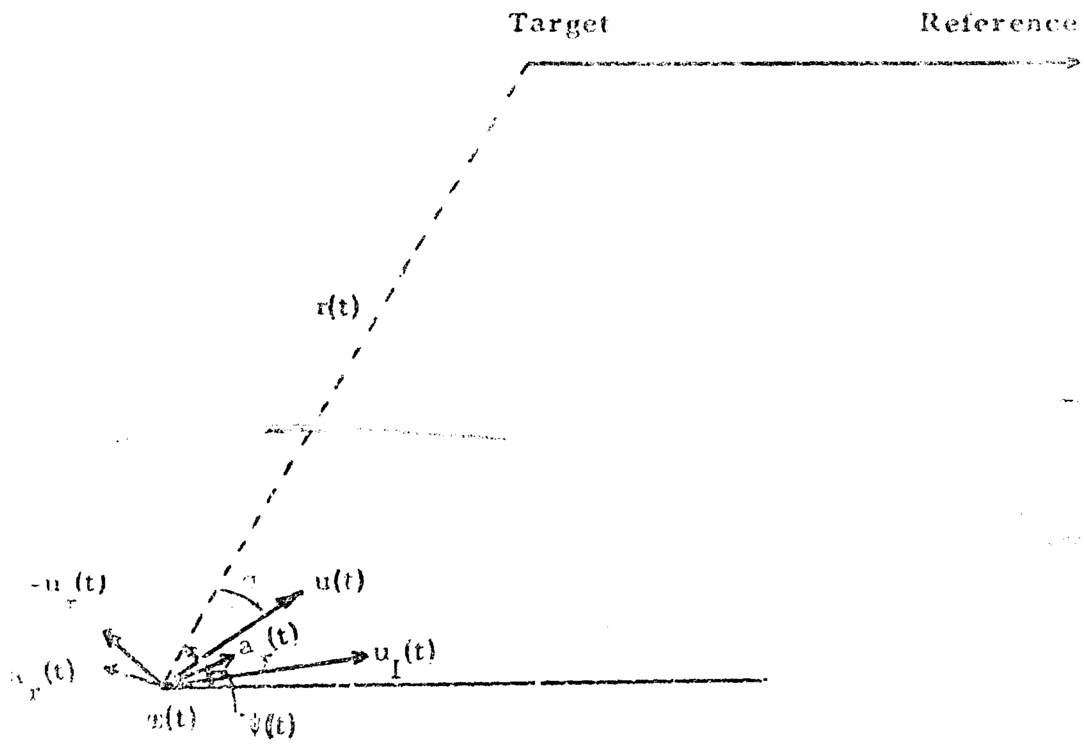

Figure 1

$\omega(t)$ is the location of the interceptor at time t

$v_I(t)$ is the velocity vector of the interceptor

$v_T(t)$ is the velocity vector of the target referred to $\omega(t)$

$a_I(t)$ is the interceptor acceleration vector

$a_T(t)$ is the target acceleration vector referred to $\omega(t)$

$r(t)$ is the line of sight vector

$v(t) = v_I(t) + v_T(t)$.

The motion of the interceptor during the interval $[t, t + \delta t]$ is shown in Figure 2. The relevant acceleration vectors can be decomposed into a component along $v(t)$ and a component normal to $v(t)$. These components will be denoted by the additional subscripts v and T respectively.

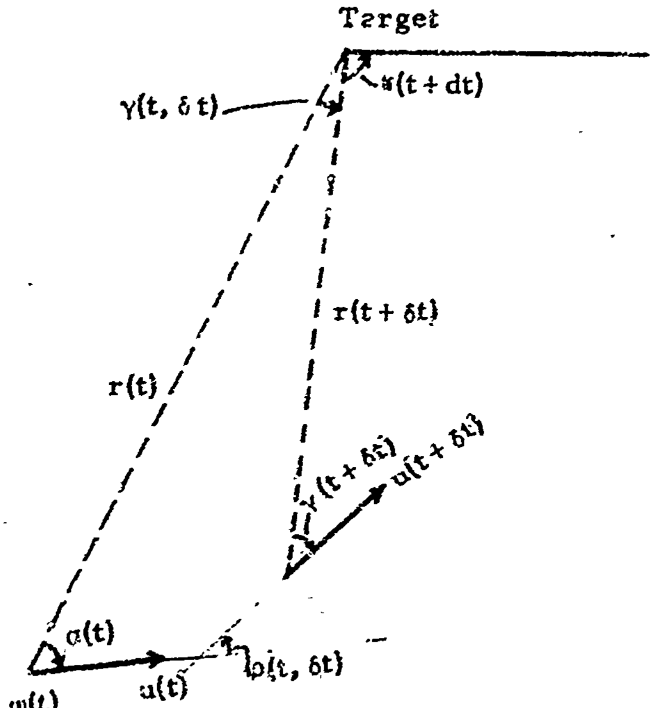

Figure 2

From Figure 2 it is evident that

$$\sigma(t+\delta t) = \sigma(t) - \gamma(t, \delta t) + \rho(t, \delta t) \tag{1}$$

In order to obtain an explicit expression for the evolution in time of $\sigma(t)$, the auxiliary variables $\rho(t, \delta t)$ and $\gamma(t, \delta t)$ must be eliminated from Eq. (1). Consider first $\gamma(t, \delta t)$. Since $\| \omega(t) - \omega(t+\delta t) \| = 0(\delta t)$[1], we have

---

[1] $\| x \|$ is the Euclidean norm of the vector x in the plane,

$$\lim_{\delta t \to 0} \frac{0(\delta t)}{\delta t} = c \neq 0$$

$$\lim_{\delta t \to 0} \frac{o(\delta t)}{\delta t} = 0 .$$

$$\gamma(t, \delta t) = - \frac{\|v(t)\| \sin \sigma(t)}{\|r(t)\|} \delta t + o(\delta t) . \qquad (2)$$

Similarly,

$$\rho(t, \delta t) = \frac{a_{IN}(t) + a_{TN}(t)}{\|v(t)\|} \delta t + o(\delta t) . \qquad (3)$$

If Eqs. (2) and (3) are substituted into Eq. (1), the following equation results

$$\sigma(t+\delta t) = \sigma(t) + \frac{\|v(t)\| \sin \sigma(t)}{\|r(t)\|} \delta t + \frac{a_{IN}(t) + a_{TN}(t)}{\|v(t)\|} \delta t + o(\delta t) \qquad (4)$$

Dividing Eq. (4) through by δt and taking the limit $\delta t \to 0$ yields

$$\dot{\sigma}(t) = \frac{\|v(t)\|}{\|r(t)\|} \sin \sigma(t) + \frac{a_{IN}(t) + a_{TN}(t)}{\|v(t)\|} . \qquad (5)$$

Equation (5) is the basic equation describing the dynamical properties of the system. Note that only those components of vehicle acceleration perpendicular to the resultant velocity vector $v(t)$ affect the line-of-sight angular rate.

III. DERIVATION OF THE OPTIMAL CONTROL LAW

It will be supposed in this section that the interceptor can orient its acceleration vector in any arbitrary direction. Since we will mechanize this system with feedback, the observable data must be specified. Assume that the directional information available on board consists of the line-of-sight direction and the interceptor velocity and acceleration vectors (these latter may be obtained by using a set of accelerometers and integrators or by velocity measurements with respect to an inertial reference system fixed in space.)

To be more precise, denote the observed portion of the vehicle dynamic characteristics by $\xi(t)$. Since it is not reasonable to presume that such time functions as line-of-sight angle can be measured without error, an additive noise vector n(t) will be added to the observations to obtain the vector $\eta(t)$, the feedback signal to the controller.

$$\eta(t) = \xi(t) + n(t) \; . \tag{6}$$

For convenience in notation denote the time function $\eta(\tau)$; $0 \leq \tau < t$ by $\eta_t(I)$.

To obtain an explicit expression for the optimum control rule, we will employ the dynamic programming formalism. Let J denote the magnitude of the terminal miss distance if the optimal control is used. By terminal miss distance we mean the euclidean distance between target

and interceptor at the end of the pursuit process. The scalar J is non-negative and is zero only if the interceptor hits the target. If the system state is given at time t, the terminal miss will be a random variable because of the dependence of vehicle acceleration on observation noise. The state of the interceptor may be described in terms of the line-of-sight angle, the range, and the velocity. If the expected value of the miss distance is used as the performance measure, we have

$$H = E\{J(\sigma(t), V(t), R(t)) \mid \eta_t(I)\}^{2} \tag{7}$$

This is a terminal control problem, and consequently, the incremental cost is identically zero. From Bellman's principle of optimality [4] we obtain the following recurrence formula

$$\begin{aligned} H &= E\{J(\sigma(t), V(t), R(t)) \mid \eta_t(I)\} \\ &= E\Big\{\min_{\substack{a(\tau) \\ t \le \tau < t+\Delta}} E\{J(\sigma(t+\Delta), V(t+\Delta), R(t+\Delta) \mid \eta_{t+\Delta}(I)\} \mid \eta_t(I)\Big\} . \end{aligned} \tag{8}$$

Since $a(\tau)$, $t \le \tau \le t+\Delta$ is functionally dependent on $\eta_t(I)$, we may use the identity

$$E\{E\{A \mid B\}\} = E\{A\}$$

---

2 To simplify the notation, capital letters will be used to denote the magnitude of the velocity, and range vectors.

to obtain

$$E\{J(\sigma(t), V(t), R(t)) \mid \eta_t(I)\} = \min_{\substack{a(\tau) \\ t \leq \tau < t+\Delta}} E\{J(\sigma(t+\Delta), V(t+\Delta), R(t+\Delta)) \mid \eta_t(I)\} .$$

If J possesses all necessary derivatives,

$$J(\sigma(t+\Delta), V(t+\Delta), R(t+\Delta)) = J(\sigma(t), V(t), R(t)) \tag{9}$$

$$+ \frac{\partial J}{\partial \sigma(t)} \frac{d\sigma}{dt} \Delta + \frac{\partial J}{\partial V(t)} \frac{dV}{dt} \Delta + \frac{\partial J}{\partial R(t)} \frac{dR}{dt} \Delta + o(\Delta) .$$

Therefore,

$$0 = \min_{\substack{a(\tau) \\ t \leq \tau < t+\Delta}} E\left\{ \frac{\partial J}{\partial \sigma(t)} \dot{\sigma}(t) + \frac{\partial J}{\partial V(t)} \frac{dV(t)}{dt} + \frac{\partial J}{\partial R(t)} \frac{dR(t)}{dt} + \frac{o(\Delta)}{\Delta} \mid \eta_t(I) \right\} . \tag{10}$$

Clearly the rate of change of $R(t)$ depends only on the velocity vector $v(t)$, and is thus independent of the minimization with respect to $a(t)$. Note that

$$\frac{dV(t)}{dt} = a_{IV}(t) + a_{TV}(t)$$

$$\frac{d\sigma}{dt} = \frac{V(t)}{R(t)} \sin \sigma(t) + \frac{a_{IN}(t) + a_{TN}(t)}{V(t)}$$

Consequently, we must seek the minimum of

$$E\left\{ \frac{\partial J}{\partial \sigma} \left[\frac{V(t)}{R(t)} \sin \sigma(t) + \frac{a_{IN}(t) + a_{TN}(t)}{V(t)} \right] \right.$$

$$\left. + \frac{\partial J}{\partial V} \left[a_{IV} + a_{TV} \right] \;\Big|\; \eta_t(I) \right\} .$$

Only a_I may be chosen at our discretion; that is we must consider

$$E\left\{ \frac{\partial J}{\partial \sigma} \frac{a_{IN}(t)}{V(t)} + \frac{\partial J}{\partial V} a_{IV} \;\Big|\; \eta_t(I) \right\}$$

subject to the constraint that the interceptor is bounded

$$a_{IN}(t)^2 + a_{IV}^2 \leq c^2$$

The interceptor acceleration is functionally dependent upon $\eta_t(I)$, and may therefore be factored out of the conditional expectation. The optimal control lies either on the boundary of the constraint set or it lies in the interior. In the former case the Lagrange multiplier technique may be used to evaluate $a_I(t)$. We must minimize

$$a_{IN} \; E\left\{ \frac{\partial J}{\partial \sigma} \frac{1}{V(t)} \;\Big|\; \eta_t(I) \right\} + a_{IV} \; E\left\{ \frac{\partial J}{\partial V} \;\Big|\; \eta_t(I) \right\}$$

$$+ \frac{\lambda}{2} \left(a_{IN}^2(t) + a_{IV}^2 \right)$$

subject to the magnitude constraint. The acceleration vector must then satisfy the equations

$$\lambda\, a_{IN}(t) + E\left\{\frac{\partial J}{\partial \sigma}\frac{1}{V(t)} \mid \eta_t(I)\right\} = 0$$

$$\lambda\, a_{IV}(t) + E\left\{\frac{\partial J}{\partial V} \mid \eta_t(I)\right\} = 0$$

From the constraint equation

$$\frac{E\left\{\frac{\partial J}{\partial V} \mid \eta_t(I)\right\}^2 + E\left\{\frac{1}{V(t)}\frac{\partial J}{\partial \sigma} \mid \eta_t(I)\right\}^2}{\lambda^2} = c^2 \tag{11}$$

Consequently,

$$a_{IN} = \frac{c\; E\left\{\frac{\partial J}{\partial \sigma}\frac{1}{V(t)} \mid \eta_t(I)\right\}}{\left[E\left\{\frac{1}{V(t)}\frac{\partial J}{\partial \sigma} \mid \eta_t(I)\right\}^2 + E\left\{\frac{\partial J}{\partial V(t)} \mid \eta_t(I)\right\}^2\right]^{1/2}}$$

$$a_{IV} = \frac{c\; E\left\{\frac{\partial J}{\partial V} \mid \eta_t(I)\right\}}{\left[E\left\{\frac{1}{V(t)}\frac{\partial J}{\partial \sigma} \mid \eta_t(I)\right\}^2 + E\left\{\frac{\partial J}{\partial V(t)} \mid \eta_t(I)\right\}^2\right]^{1/2}} \tag{12}$$

The optimal control will lie on the interior of the constraint set only if

$$E\left\{\frac{\partial J}{\partial \sigma} \mid \eta_t(I)\right\} = E\left\{\frac{\partial J}{\partial V} \mid \eta_t(I)\right\} = 0 \tag{13}$$

It seems reasonable to suppose that the set of $t \in [0, T]$ satisfying Eq. (13) is finite or at least countable. and for this reason such points will be neglected in this investigation.

IV. OPTIMAL CONTROL LAW WITH DIRECTION CONSTRAINT

Equation (12) gives an explicit expression for the optimal control law. Two major drawbacks to actual implementation of this controller become immediately evident. On the one hand, the partial differential equation for the conditional expectation for J (see Eq. (10)) must be solved. Because of the bound on the control, this is a far from trivial task. Secondly, the interceptor must have some means of sensing the direction of its velocity vector. For some vehicles this would involve an unwarranted increase in controller complexity.

One way to obviate these difficulties is to fix the direction of the interceptor acceleration with respect to a measured direction. Since the direction of the line-of-sight is readily available, let us suppose that the acceleration vector a(t) is orthogonal to the line-of-sight vector. Clearly, this additional restriction on the acceleration can only increase the performance measure studied in the previous section. The justification of the restriction lies in the reduced system complexity obtainable.

With the direction constraint

$$a_{IN}(t) = a(t) \cos \sigma(t)$$

$$a_{IV}(t) = a(t) \sin \sigma(t)$$

$$|a(t)| \leq c$$

We must choose a(t) to minimize

$$E\left\{ \frac{\partial J}{\partial \sigma} a(t) \frac{\cos \sigma(t)}{V(t)} + \frac{\partial J}{\partial V(t)} a(t) \sin \sigma(t) \mid \eta_t(I) \right\}$$

Since a(t) is functionally dependent on $\eta_t(I)$, the optimal control is

$$a(t) = -c \operatorname{sgn} E\left\{ \frac{\partial J}{\partial \sigma} \frac{\cos \sigma(t)}{V(t)} + \frac{\partial J}{\partial V(t)} \sin \sigma(t) \right\} \tag{14}$$

If this value of a(t) is placed in Eq. (10), a partial differential equation for the conditional expectation of J would result. Solving this equation would represent quite a task and no attempt will be made to solve the equation explicitly here. Two intuitively evident conjectures about properties of J, however, do permit us to obtain the optimal control rule. Recalling that J is a non-negative function for all values of its arguments, we might suppose that $J(\sigma(t), V(t), R(t))$ is an increasing function of $\sigma(t)$ for all positive V(t) and R(t) if $\frac{\pi}{2} > \sigma(t) > 0$ and a decreasing function of $\sigma(t)$ for all positive V(t) and R(t) if $-\frac{\pi}{2} < \sigma(t) < 0$. Then

$$\operatorname{sgn} \frac{\partial J}{\partial \sigma(t)} = \operatorname{sgn} \sigma(t) .$$

Furthermore, V(t) is always a positive quantity and Eq. (5) indicates that an increase in V(t) has the effect of reducing the influence of the interceptor acceleration. Consequently, it seems reasonable to suppose that

$$\frac{\partial J}{\partial V(t)} > 0$$

for all positive R(t) and all $\sigma(t)$.

From the symmetry of the problem

$$J(\sigma(t), V(t), R(t)) = J(-\sigma(t), V(t), R(t)) .$$

Therefore, we have the result that

$$\frac{\partial J}{\partial \sigma(t)} \frac{\cos \sigma(t)}{V(t)} + \frac{\partial J}{\partial V(t)} \sin \sigma(t)$$

is an odd function of $\sigma(t)$. In fact, if the conditional probability distribution of $\sigma(t)$ is symmetric, and if $|\sigma(t)| \leq \pi/2$,

$$\operatorname{sgn} E\left\{ \frac{\partial J}{\partial \sigma(t)} \frac{\cos \sigma(t)}{V(t)} + \frac{\partial J}{\partial V(t)} \sin \sigma(t) \,|\, \eta_t(I) \right\} = \operatorname{sgn} E\left\{ \sigma(t) \,|\, \eta_t(I) \right\} . \quad (15)$$

Substituting this result into Eq. (14)

$$a(t) = -c \operatorname{sgn} E\left\{ \sigma(t) \mid \eta_t(I) \right\}. \quad (16)$$

V. CHOICE OF REQUIRED MEASUREMENTS

Equation (16) provides the optimal control rule and for its implementation we have to determine $\operatorname{sgn} E\{\sigma(t) \mid \eta_t(I)\}$. However, $\sigma(t)$ cannot be measured directly since (see Figure 1) $v_T(t)$ is an unknown quantity to the interceptor. From Figure 2

$$\Psi(t + \alpha t) = \Psi(t) - \gamma(t, \delta t) \quad (17)$$

Replacing $\gamma(t, \delta t)$ by its value in Eq. (2), directing both sides by $\delta(t)$ and taking the limit as $t \to 0$ we have

$$\dot{\Psi}(t) = -\frac{\|u(t)\|}{\|r(t)\|} \sin \sigma(t) \quad (18)$$

Since $\|v(t)\|$ and $\|r(t)\|$ are positive quantities and $|\sigma(t)| < \pi/2$ then

$$\operatorname{sgn} \sigma(t) = \operatorname{sgn} \sin \sigma(t) = -\operatorname{sgn} \dot{\Psi}(t) \quad (19)$$

Consequently, the quantity which should be measured is the line of sight rotation with respect to an inertial coordinate system. This quantity can be measured on-board by means of a tracking system. Finally, the

optimal control rule can be written as

$$u(t) = + c \operatorname{sgn} E \left\{ \dot{\Psi}(t) \mid \eta_t(I) \right\} . \qquad (20)$$

Consequently, $\xi(t)$ in (6) can be taken as $\dot{\Psi}(t)$ and then

$$\eta(t) = \dot{\Psi}(t) + n(t) ,$$

where $n(t)$ is the measurement noise. When statistics of $\dot{\Psi}(t)$ and $n(t)$ are known, the estimation of $E\left\{\dot{\Psi}(t) \mid \eta(I)\right\}$ can be carried out by appropriate optimal filtering [5].

A different procedure in addition to one discussed above can be used in the special case when the target is stationary. In this case $v(t) = v_I(t)$. The angle between $v_I(t)$ and an axis fixed with respect to the vehicle body can be measured by means of appropriate instruments on board. Furthermore, the angle between $r(t)$ and the body fixed axis can be measured by means of an antenna system fixed with respect to the vehicle. The desired angle $\sigma(t)$ is the difference between these two angles. The advantage of this approach is that it is not necessary to mount the tracking antenna on a stable platform.

VI. ADDITIONAL SYSTEM DYNAMICS

In practice there may be a delay time of d seconds between a value of $\dot{\Psi}(t)$ and generation of the appropriate acceleration given by (20). The delay time d may be actually an approximation to various system dynamics involved such as the tracking loop dynamics, reaction jets, etc. It is easily seen that under this condition the optimum control rule given by (20) should be modified to

$$A(t) = +c \operatorname{sgn} E\left\{\dot{\Psi}(t) \mid \eta_{t-d}(I)\right\}$$

where $\eta_{t-d}(I)$ represents the time function $\eta(\tau)$; $0 \leq \tau \leq t-d$.

In other words the control is based on the estimate of $\dot{\Psi}(t)$ corresponding to the data received over the interval $0 \leq \tau \leq t-d$.

VII. CONCLUSIONS

The optimum control law for a vehicle which seeks to reach a moving target has been derived where the performance criterion is the expectation of the terminal miss distance and the thrust vector is bounded in magnitude. It is shown that the optimum control law is a function of the sign of the expected value of the line-of-sight angular rate with respect to an inertial coordinate system.

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13. ABSTRACT

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